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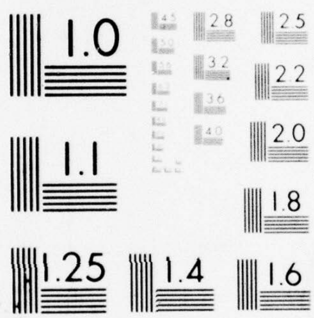

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PERFORMANCE BOUNDS ON SPREAD SPECTRUM

MULTIPLE ACCESS COMMUNICATION SYSTEMS

by

Kung Yao

TECHNICAL REPORT No. 1

October 1976

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PERFORMANCE BOUNDS ON SPREAD SPECTRUM MULTIPLE ACCESS COMMUNICATION SYSTEMS

By KUNG YAO

Summary. -Several approaches for the evaluation of upper and lower bounds on error probability of spread spectrum multiple access communication systems are presented. These bounds are obtained by utilizing an isomorphism theorem in the theory of moment spaces. From this theorem, we generate closed, compact, and convex bodies, where one of the coordinates represents error probability, while the other coordinate represents a generalized moment of the multiple access interference random variable. Derivations for the second moment, fourth moment, single exponential moment, and multiple exponential moment are given in terms of the partial cross correlations of the codes used in the system.

Introduction. -Spread spectrum multiple access technique is of use in a multi-user computer communication network system [1] as well as in a more conventional satellite communication system with a single wide-band repeater [2]. In such situations, a code modulation spread spectrum multiple access (SSMA) system is considered suitable for a network of low-cost mobile ground based and airbourned users requiring no network control. In any case, for this and related SSMA systems, the exact evaluation of error probability has been considered a formidable task. Error probability obtained by complete simulation of such systems may involve considerable computational cost.

In this paper, we present several approaches based on the theory of moment spaces to obtain upper and lower bounds on the error probability of a SSMA system. As expected, bounds that use moments that require more computational effort are generally tighter than those that require less. As to be seen, the second moment, fourth moment, single exponential moment, and multiple exponential moment require increasing computational effort. Indeed, by taking a sufficiently large number of terms in the multiple exponential moment case, we can make the upper and lower bounds arbitrarily tight.

SSMA Model. -There are various forms of SSMA communications systems. In a direct code modulation SSMA system, the data stream of each user modulates a shift-register (SR) generated sequence code to obtain the spread spectrum effect. The multiple access capability is achieved by requiring each user's code word to be near orthogonal. In this paper, we shall only consider the model of an asynchronous SSMA system as discussed in [3].

Thus, we allow the time delay's τ_i and phase angles θ_i of different users to be r.v.'s. The input to each receiver consists of the sum of all K users' signals and additive white Gaussian noise. Each receiver consists of a matched filter matched to its corresponding code word. Without loss of generality, we consider the first receiver. Then we assume it is completely synchronized to its own code word. Thus, $\theta_1 = \tau_1 = 0$. But $\theta_2,\ldots,\theta_K$ and $\tau_2,\ldots,\tau_K$ are independent and uniformly distributed r.v.'s. Thus, the output of the matched filter of the 1st receiver is given by

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The author is with the University of California, Los Angeles.

$$y = \sqrt{\frac{P}{2}} \int_0^T a_1^2(t) b_1(t) dt$$

$$+ \sqrt{\frac{P}{2}} \sum_{i=2}^{K} \int_0^T a_i(t-\tau_i) b_i(t-\tau_i) a_1(t) dt \cos\theta_i$$

$$+ \int_0^T n(t) a_1(t) \cos w_c t dt = hb_{1,0} + Z + n \ . \tag{1}$$

The first term of y represents the desired signal, the second term, Z, represents the interference from the other $(K-1)$ users, and the last term, n, is a Gaussian r.v. of zero-mean and variance σ^2. All users are assumed to have equal power P. Here the information data of the i-th user is defined by

$$b_i(t) = \sum_{n=-\infty}^{\infty} b_{i,n} P_T(t-nT) \ ,$$

where the $b_{i,n}$'s are i.i.d. r.v.'s taking values $+1$ and -1 with equal probability, and $P_T(\cdot)$ is a unit height rectangular window from 0 to T. The code waveform is defined by

$$a_i(t) = \sum_{j=-\infty}^{\infty} a_j^{(i)} P_{T_c}(t-jT_c) \ ,$$

where $a_j^{(i)}$ is the SR code sequence of the i-th user and consists of $+1$ and -1 which is periodic with period p and of chip length T_c.

Now, consider the error probability of the first user, assuming all K user's code words have been specified. Denote this error probability by P_e. Then

$$P_e = \frac{1}{2} \Pr\{h+Z+n<0\} + \frac{1}{2} \Pr\{-h+Z+n>0\}$$

$$= E\{[Q\left(\frac{h+|Z|}{\sigma}\right) + Q\left(\frac{h-|Z|}{\sigma}\right)]/2\} \tag{2a}$$

$$= E\{Q\left(\frac{h+Z}{\sigma}\right)\} \ , \tag{2b}$$

where

$$Q(x) = (2\pi)^{-\frac{1}{2}} \int_x^{\infty} \exp(-t^2/2) dt \ .$$

P_e in (2a) or (2b) is expressed as a generalized moment where the expectation operation, E, is taken over all the r.v.'s τ_i, $b_{i,-1}$, $b_{i,0}$, and θ_i in Z. As to be seen, depending on specific cases, sometimes we prefer to use P_e given by (2a), while other times we prefer to use P_e given by (2b). In general, since Z is extremely complicated, it is not possible to evaluate this moment analytically. However, suppose it is possible to evaluate some other moments of Z. Then if we can find relationships among the moments defined by P_e and other moments that we can evaluate readily, then we can obtain information about the error probability. In the next section, we shall state an isomorphism theorem from the theory of moment spaces that shall yield a precise geometric interpretation of this concept.

Moment Space Error Bounds. -The following theorem which was originally developed in the theory of games, shall provide relationships among arbitrary moments of a random variable.

Let Z be a random variable with a probability distribution function $G_Z(z)$ defined over a finite closed interval $I = [a,b]$. Let $k_1(z), k_2(z), \ldots, k_N(z)$ be a set of N continuous functions defined on I The generalized moment of the random variable Z induced by the function $k_i(z)$ is

$$m_i = \int_I k_i(z) dG_Z(z) = E_Z\{k_i(z)\}, \quad i = 1, \ldots, N \tag{3}$$

We denote the N-th moment space $\mathcal{M}$ as

$$\mathcal{M} = \{\underline{m} = (m_1, \ldots, m_N) \in \mathbb{R}^N | m_i = \int_a^b k_i(z) dG_Z(z) \ , \ i = 1, \ldots, N\}$$

where G_Z ranges over the set of probability distributions defined on $I = [a,b]$ and $\mathbb{R}^N$ denotes N-dimensional Euclidean space. Then $\mathcal{M}$ is a closed, bounded, and convex set. Now let $\mathcal{C}$ be the curve $\underline{r} = (r_1, \ldots, r_N)$ traced out in $\mathbb{R}^N$ by $r_i = k_i(z)$ for z in I . Let $\mathcal{H}$ be the convex hull of $\mathcal{C}$. Then

$$\mathcal{H} = \mathcal{M} . \tag{4}$$

This theorem has been used in [4] to bound P_e encountered in intersymbol interference problems with $N = 2$. In order to explain the application of this theorem in obtaining P_e bounds, consider a plot of $k_2(z)$ versus $k_1(z)$. Now, we take $k_2(z)$ to be the expression inside the curly bracket given either by (2a) or (2b). Thus, $m_2 = E\{k_2(z)\} = P_e$. We shall consider several different $k_1(z)$. In any case, $k_1(z)$ will be chosen such that, $m_1 = E\{k_1(z)\}$ is evaluable. When we plot specific $k_2(z)$ versus $k_1(z)$, the curve $\mathcal{C}$ typically turns out to be given by parts of the curve shown in Figure 1. The curve in Figure 1 consists of several sections that are convex $\cup$ or $\cap$ functions. Let the points C , E , G , I , etc. be points of infection of the curve. Then curve ABC is convex $\cup$, CDE is convex $\cap$, EFG is convex $\cup$, GHI is convex $\cap$, etc. Suppose a plot of $k_2(z)$ versus $k_1(z)$ yields the curve $\mathcal{C}$ given by ABB' . Then the upper envelope of this body $\mathcal{H}$, is given by the straight line AB' , while the lower body is given just by ABB' . Thus, from the above theorem and (4), we can obtain upper and lower bounds on P_e . Suppose $E\{k_1(z)\}$ yields a number m_1 (which has to be less or equal to $k_1(z_{B'})$) . Then the lower bound is given by $P_e^{(L)} = k_2(k_1^{-1}(m_1))$ while the upper bound is given by $P_e^{(U)} = k_2(z_A) + (m_1 - k_1(z_A)) \times \{(k_2(z_{B'}) - k_2(z_A))/(k_1(z_{B'}) - k_1(z_A))\}$. Here, we used the notation of $A = (k_1(z_A), \ k_2(z_A))$, $B = (k_1(z_B) \ , \ k_2(z_B))$, etc.

Now, suppose a different $k_2(z)$ versus $k_1(z)$ yields a curve $\mathcal{C}$ given by ABCD . Thus, parts of $\mathcal{C}$ is convex $\cup$ while parts of $\mathcal{C}$ is convex $\cap$. Then the first part of the upper envelope of $\mathcal{H}$, is given by the straight line AC' , while the second part is given by the curve C'D . The point C' is defined by equating the slope of the chord AC' to the derivation of the curve $\mathcal{C}$ at c' . That is,

$$\frac{k_2(z_{c'}) - k_2(z_A)}{k_1(z_{c'}) - k_1(z_A)} = \frac{k_2'(z_{c'})}{k_1'(z_{c'})} . \tag{5}$$

We note, since $k_2(z)$, $k_2'(z)$, $k_1(z)$, $k_1'(z)$, and z_A are known,

we can solve for $z_{C'}$ in (5) easily by using any single root solution technique (i.e. Newton method, Regula Falsi Method, etc.). Thus, if $k_1(z_A) \le m_1 \le k_1(z_{C'})$, we have $P_e^{(U)} = k_2(z_A) + (m_1 - k_1(z_A)) \times \{(k_2(z_{C'}) - k_2(z_A))/(k_1(z_{C'}) - k_1(z_C))\}$. If $k_1(z_{C'}) \le m_1 \le k_1(z_D)$, we have $P_e^{(U)} = k_2(k_1^{-1}(m_1))$. By similar arguments, the lower envelope of $\mathscr{K}$, is given by the curve ABB" and the straight line B"D. The point B" or $Z_{B''}$ can be obtained from

$$\frac{k_2(z_D) - k_2(z_{B''})}{k_1(z_D) - k_1(z_{B''})} = \frac{k_2'(z_{B''})}{k_1'(z_{B''})} . \tag{6}$$

Thus, if $k_1(z_A) \le m_1 \le k_1(z_{B''})$, we have $P_e^{(L)} = k_2(k_1^{-1}(m_1))$. If $k_1(z_{B''}) \le m_1 \le k_1(z_D)$, then we have $P_e^{(L)} = k_2(z_{B''}) + (m_1 - k_1(z_{B''})) \times \{(k_2(z_D) - k_2(z_{B''}))/(k_1(z_D) - k_1(z_{B''}))\}$.

Another interesting situation is when $\mathscr{C}$ is given by or contains the curve CDEFGHI. Then the upper envelope is given by the curve CD', the straight line D'H and the curve HI. In particular, the points D' (or $z_{D'}$) and H (or z_H) are obtained from

$$\frac{k_2(z_H) - k_2(z_D)}{k_1(z_H) - k_1(z_D)} = \frac{k_2'(z_D)}{k_1'(z_D)} = \frac{k_2'(z_H)}{k_1'(z_H)} . \tag{7}$$

From (7), we can try to obtain the two unknowns z_D and z_H from two non-linear equations. In practice, because of the local convex ∪ properties of the curve near D' and H, we can use iterative solution approach to find z_D and z_H quite readily. Thus, for $k_1(z) \le m_1 \le k_1(z_{D'})$, $P_e^{(U)} = k_2(k_1^{-1}(m_1))$, for $k_1(z_{D'}) \le m_1 \le k_1(z_H)$, $P_e^{(U)} = k_2(z_{D'}) + (m_1 - k_1(z_{D'})) \times \{(k_2(z_H) - k_2(z_{D'}))/(k_1(z_H) - k_1(z_{D'}))\}$; for $k_1(z_H) \le m_1 \le k_1(z_I)$, $P_e^{(U)} = k_2(k_1^{-1}(m_1))$. The evaluation of lower envelopes and lower error bounds are similar as that for upper envelopes and upper error bounds.

From all the above discussions, it is clear that given any $k_2(z)$ and $k_1(z)$, explicit evaluation of upper and lower error bounds are possible as soon as the moment $m_1 = E\{k_1(Z)\}$ and the domain of z are available.

Evaluation of Moments and Maximum Distortion. —Now, we consider the explicit evaluations of several different moments $m_1 = E\{h_1(Z)\}$ as well as the maximum distortion D.

In order to use $m_1 = E\{Z^2\}$, let $k_1(z) = z^2$ and $k_2(z)$ be given by expression (2a). Then the domain of $k_1(z)$ is $[0,D]$, when $D = \text{Max } Z$. From the definition of the r.v. Z in (1), it is clear that $E[Z] = 0$. From (19) of [3], it is seen that for fixed $b_{1,0}$,

$$m_1 = E[Z^2] = \frac{PT_c^3}{12T} \sum_{i=2}^{K} r_{i1} , \tag{8}$$

where

$$r_{ij} = \sum_{n=0}^{N-1} \{\rho_{ij}^2(n)+\rho_{ij}(n)\rho_{ij}(n+1)+\rho_{ij}^2(n+1)+\beta_{ij}^2(n)+\beta_{ij}(n)\beta_{ij}(n+1)$$

$$+ \beta_{ij}^2(n+1)\} , \tag{9}$$

$$\rho_{ij}(n) = \sum_{m=0}^{n-1} a_{m-n}^{(i)} a_m^{(j)} , \tag{10}$$

$$\beta_{ij}(n) = \sum_{m=n}^{N-1} a_{m-n}^{(i)} a_m^{(j)} . \tag{11}$$

The period of the codes is denoted by p and $N = T/T_c$ is assumed to be an integral multiple of p.

Now, consider the evaluation of D defined as the maximum value of the r.v. Z, where the maximization is considered over θ_i, τ_i, $b_{i,-1}$, and $b_{i,0}$. After some algebra, it can be seen that

$$D = \left(\frac{P}{2}\right)^{\frac{1}{2}} T_c \sum_{i=2}^{K} \max_{1 \le n_i \le N} \{|\rho_{i1}(n_i)| + |\beta_{i1}(n_i)|\} . \tag{12}$$

Thus, m_1 and D can be readily evaluated and depend on the partial cross correlations (given by (10) and (11)) of the code words chosen by the K users.

In order to use $m_1 = E\{Z^4\}$, let $k_1(z) = z^4$ and $k_2(z)$ be given by expression (2a). Then the domain of $k_1(z)$ is also $[0,D]$. In order to evaluate $m_1 = E[Z^4]$, let

$$z_i = \int_0^T a_i(t-\tau_i) b_i(t-\tau_i) a_1(t) dt \cos\theta_i = c_i \cos\theta_i , \tag{13}$$

Then

$$E[Z^4] = \left(\frac{P}{2}\right)^2 E\{\sum_{i=2}^{K} z_i\}^4 = \left(\frac{P}{2}\right)^2 \{\sum_{i=2}^{K} E[z_i^4] + 6 \sum_{i=2}^{K-1} E[z_i^2]$$

$$[\sum_{j=i+1}^{K} E[z_j^2]]\} . \tag{14}$$

Here, each $E[z_i^2]$ is in the form of (8) and can be evaluated by using (8) - (11). Now,

$$E[z_i^4] = E[\cos^4\theta_i] E[c_i^4] = \left(\frac{3}{8}\right)\left(\frac{1}{T}\right) \sum_{n=0}^{N-1} \int_0^{T_c} \{R_{i1}^4(\lambda+nT_c) + \hat{R}_{i1}^4(\lambda+nT_c)$$

$$+ 6R_{i1}^2(\lambda+nT_c)\hat{R}_{i1}^2(\lambda+nT_c)\} d\lambda . \tag{15}$$

Furthermore,

$$\begin{aligned} R_{i1}(\lambda+nT_c) &= A_n T_c + B_n \lambda \\ \hat{R}_{i1}(\lambda+nT_c) &= \hat{A}_n T_c + \hat{B}_n \lambda \end{aligned} \tag{16}$$

where

$$\begin{aligned} A_n &= \rho_{i1}(n) , \quad B_n = \rho_{i1}(n+1) - \rho_{i1}(n) \\ \hat{A}_n &= \beta_{i1}(n) , \quad \hat{B}_n = \beta_{i1}(n+1) - \beta_{i1}(n) \end{aligned} \tag{17}$$

By using (16) and (17), each $E[z_i^4]$ is given by

$$E[z_i^4] = \left(\frac{3T_c^5}{8T}\right) \sum_{n=0}^{N-1} \{(A_n^4 + \hat{A}_n^4) + 2(A_n^3 B_n + \hat{A}_n^3 \hat{B}_n) + 2(A_n^2 B_n^2 + \hat{A}_n^2 \hat{B}_n^2) + (A_n B_n^3 + \hat{A}_n \hat{B}_n^3)$$

$$+ (\tfrac{1}{5})(B_n^4+\hat{B}_n^4)+6(A_n^2\hat{A}_n^2+A_nB_n\hat{A}_n^2+\hat{A}_n\hat{B}_nA_n^2)+2(A_n^2\hat{B}_n^2+\hat{A}_n^2B_n^2+4A_nB_n\hat{A}_n\hat{B}_n)$$
$$+3(A_nB_n\hat{B}_n^2+\hat{A}_n\hat{B}_nB_n^2)+(\tfrac{6}{5})B_n^2\hat{B}_n^2\} \; . \qquad (18)$$

Thus, each term in (14) for m_1 can be evaluated.

In order to use $m_1 = E\{\exp[c(h+Z)]\}$, let $k_1(z) = \exp[c(h+Z)]$ and $k_2(z)$ be given by expression (2b). Then the domain of $k_1(z)$ is $[h-D,h+D]$. From the definition of Z in (1), we have

$$m_1 = \exp(ch) \prod_{i=2}^{K} E_{\theta_i,\tau_i,b_{i,-1},b_{i,0}}\{\exp[(c\cos\theta_i)(b_{i,-1}\times \int_0^{\tau_i} a_i(t-\tau_i)a_1(t)dt$$
$$+ b_{i,0}\times \int_{\tau_i}^{T} a_i(t-\tau_i)a_1(t)dt)]\} \qquad (19)$$

Thus, we need to consider the evaluation of the expression

$$E_{\theta_i,\tau_i,b_{i,-1},b_{i,0}}\{\cdot\}$$

for each $2 \le i \le K$.

$$E_{\theta_i,\tau_i,b_{i,-1},b_{i,0}}\{\cdot\} = E_{\theta_i,\tau_i}\{\exp[c\cos\theta_i)(R_{1i}(\tau_i)+\hat{R}_{1i}(\tau_i))]$$
$$+ \exp[(c\cos\theta_i)(R_{1i}(\tau_i)-\hat{R}_{1i}(\tau_i))]+\exp[(c\cos\theta_i)(-R_{1i}(\tau_i)+\hat{R}_{1i}(\tau_i))]$$
$$+ \exp[(c\cos\theta_i)(-R_{1i}(\tau_i)-\hat{R}_{1i}(\tau_i))]\}/4$$
$$= (1/4T)E_{\theta_i}\sum_{n=0}^{N-1}\int_0^{T_c}\{\exp[(c\cos\theta_i)((A_n+\hat{A}_n)+(B_n+\hat{B}_n)\lambda)]$$
$$+ \exp[(c\cos\theta_i)((A_n-\hat{A}_n)+(B_n-\hat{B}_n)\lambda)]+\exp[(c\cos\theta_i)((-A_n+\hat{A}_n)$$
$$+ (-B_n+\hat{B}_n)\lambda)]+\exp[(c\cos\theta_i)((-A_n-\hat{A}_n)+(-B_n-\hat{B}_n)\lambda)]\}d\lambda \; , \qquad (20)$$

where $R_{1i}(\cdot)$ and $\hat{R}_{1i}(\cdot)$ are given by (16) and A_n, B_n, $\hat{A}_n$, and $\hat{B}_n$ are given by (17). After performing the integration in (20), we have

$$E_{\theta_i,\tau_i,b_{i,-1},b_{i,0}}\{\cdot\} = (1/4N)E_{\theta_i}\sum_{n=0}^{N-1}\sum_{m=1}^{4}\{[\exp(\alpha_i^{(m)}(n)\cos\theta_i$$
$$- \exp(\beta_i^{(m)}(n)\cos\theta_i)]/[\alpha_i^{(m)}(n)-\beta_i^{(m)}(n)]\cos\theta_i\}$$

where

$$\alpha_i^{(1)}(n) = cT_c(\rho_{1i}(n+1)+\hat{\rho}_{1i}(n+1)) = -\alpha_i^{(4)}(n)$$
$$\alpha_i^{(2)}(n) = cT_c(\rho_{1i}(n+1)-\hat{\rho}_{1i}(n+1)) = -\alpha_i^{(3)}(n)$$

$$\beta_i^{(1)}(n) = cT_c(\rho_{i1}(n) + \hat{\rho}_{i1}(n)) = -\beta_i^{(4)}(n)$$
$$\beta_i^{(2)}(n) = cT_c(\rho_{i1}(n) - \hat{\rho}_{i1}(n)) = -\beta_i^{(3)}(n) \ . \tag{22}$$

From an integral representation of $I_0(z)$ given by (8.431.1) in [5], we obtain

$$2\pi \int_\beta^\alpha I_0(z)dz = 2\int_{-1}^{1} \{[\exp(\alpha t) - \exp(\beta t)]/t(1-t^2)^{\frac{1}{2}}\}dt$$
$$= \int_0^{2\pi} \{[\exp(\alpha\cos\theta) - \exp(\beta\cos\theta)]/\cos\theta\}dt \tag{23}$$

Thus, substituting (23) in (21) and noting $I_0(z)$ is an even function,

$$E_{\theta_i,\tau_i,b_{i,-1},b_{i,0}}\{\cdot\} = (1/2N)\sum_{n=0}^{N-1}\sum_{m=1}^{2}[1/(\alpha_i^{(m)}(n)-\beta_i^{(m)}(n)] \times \int_{\beta_i^{(m)}(n)}^{\alpha_i^{(m)}(n)} I_0(z)dz \tag{24}$$

Thus, substituting (24) in (19), we obtain the single exponential moment,

$$m_1(c) = \exp(ch)\prod_{i=2}^{K}\{(1/2N)\sum_{n=0}^{N-1}\sum_{m=1}^{2}[1/(\alpha_i^{(m)}(n)-\beta_i^{(m)}(n))] \times \int_{\beta_i^{(m)}(n)}^{\alpha_i^{(m)}(n)} I_0(z)dz\} \tag{25}$$

where $\alpha_i^{(m)}(n)$ and $\beta_i^{(m)}(n)$ are defined in (22). These expressions are given in terms of the partial cross correlations of the codes, $\rho_{i1}(n)$ and $\hat{\rho}_{i1}(n)$, defined in (10) and (11).

Finally, let

$$k_1(z) = \sum_{j=1}^{J} d_j \exp[c_j(h+z)] \ , \tag{26}$$

where d_j and c_j are real-valued numbers, and $k_2(z)$ be given by expression (2b), then the multiple exponential moment is given by

$$m_1 = \sum_{j=1}^{J} d_j m_1(c_j) \ , \tag{27}$$

where $m_1(c_j)$ is given by (25) with $c = c_j$.

Conclusions. -In the previous section, we derived the second moment, fourth moment, single exponential moment, and multiple exponential moment for the spread spectrum multiple access system. These moments are given in (8), (14), (25), and (27), respectively. Clearly, the computational efforts involved in evaluating these moments are of increasing order. However, in general, we can show the resulting upper and lower error bounds become tighter. In the last two cases, we assume the parameters c , c_j ,

and d_j in (25) and (27) are selected appropriately.

There are several approaches in obtaining explicit moment space error bounds. From our earlier discussions, it is clear that by plotting $k_2(z)$ versus $k_1(z)$ graphically and then finding its convex hull, we can obtain the upper and lower error bound immediately. Indeed, we have used computer-controlled plotter and obtained satisfactory bounds. Furthermore, this approach is applicable to any $k_2(z)$ and $k_1(z)$ functions. An alternative approach is clearly the use of analytical solution to the upper and lower envelopes of the convex body. Bounds based on the second moment are discussed and given in [4], [7], and [8]. Similar arguments in [4] and [6] lead to bounds based on the fourth moment. Various possible error bounds and associated regions of c for the single exponential moment are given in [4]. In particular, the selection of the optimum values of c for the upper and the lower bounds are treated. Finally, for systems with large maximum distortion D, the range of possible values of $k_2(z)$ is large. In order to obtain tight error bounds, $k_1(z)$ must approximate $k_2(z)$ closely. In (26), by using large numbers of terms J with proper d_j and c_j's, we can make $k_1(z)$ close to $k_2(z)$. Unfortunately, the explicit solution of this approximation problem for d_j and c_j with finite J is unknown for Chebychev norm [8]. From an efficiency point of view for a fixed J, this norm is the most appropriate one to use for minimizing the distance between the upper and lower envelopes. However, in L_2 norm, it is easy to select a set of c_j's such that $k_1(z)$ converges to $k_2(z)$ as J becomes unbounded. Then $k_1(z)$ can approximate $k_2(z)$ arbitrarily well for some finite J. Thus, the associated upper and lower bounds using m_1 given by (27) can be made arbitrarily tight for some finite J.

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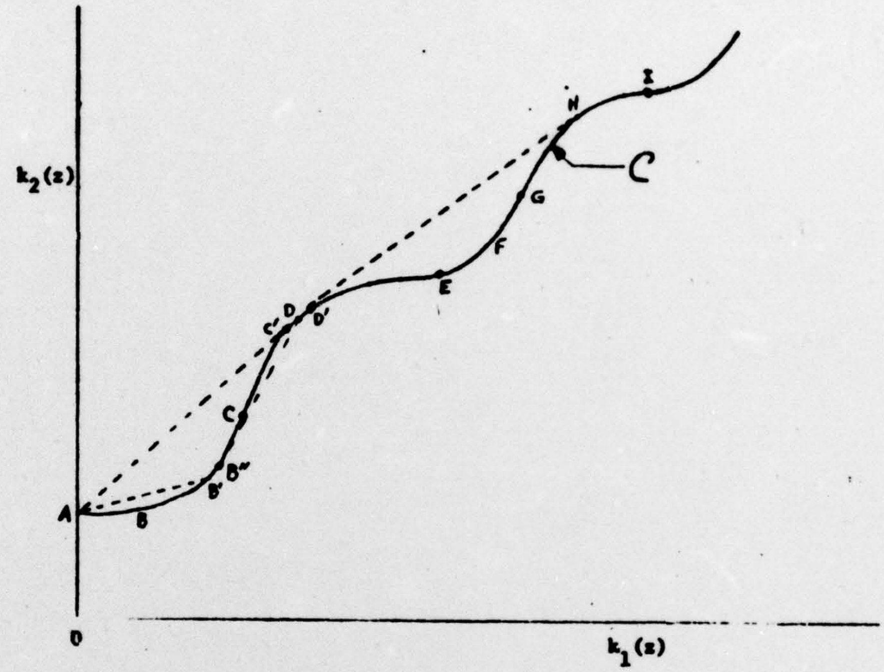

Figure 1. $k_2(z)$ versus $k_1(z)$